No-arbitrage valuation fails in arbitrage-free complete markets

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Abstract

With the only source of uncertainty being an underlying asset market, it is shown for a system of financial firms that – given any state of the market – the equilibrium maturity prices of what are usually considered typical contingent claims are not necessarily uniquely determined if circular financial interdependence exists. No-arbitrage valuation of such claims before maturity therefore cannot work – neither by replication, nor by risk-neutral pricing – even if the underlying market is arbitrage-free and every state-contingent claim can be replicated. This valuation problem is fundamentally different from the one for incomplete markets. Four examples involving equity holdings, puts, calls, binary options, bear spreads, and contingent convertible bonds are provided. Specifically binary derivatives are problematic, as they are prone to not adhering to a convexity condition for derivatives which is shown to be sufficient to avoid price indeterminacy. The failure of no-arbitrage valuation can also affect simple debt or equity, and spread to other parts of a financial system. Ignorance of this problem could cause or worsen systemic price uncertainty during financial crises.

Key words: Asset valuation, binary options, complete markets, contingent convertible bonds, cross-ownership, derivatives pricing, financial interdependence, financial networks, financial regulation, multiple equilibria, no-arbitrage pricing, structural model, systemic risk.

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1 Introduction

If a student on a finance course was asked whether any claim in an arbitrage-free complete financial market had a uniquely determined no-arbitrage price, the answer would possibly

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